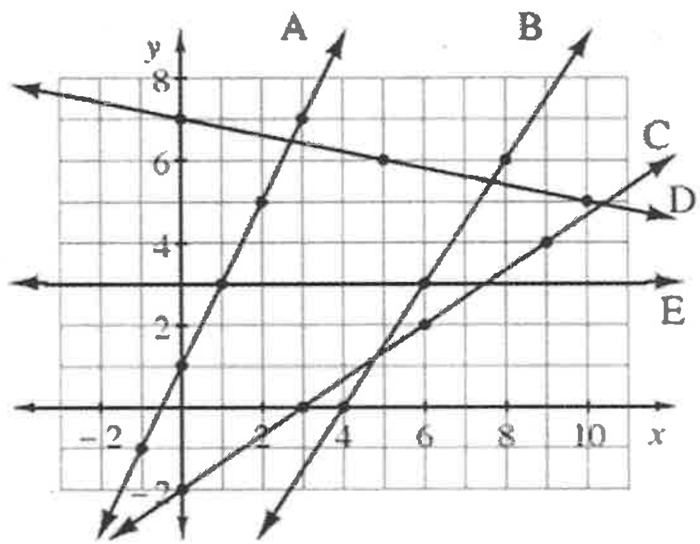
2-24. Find the graph shown at right

2-24.



a. Which is the steepest line? Which is steeper, line B or line C?

b. Draw slope triangles for lines A, B, C, D, and E using the highlighted points on each line. Label Δx and Δy for each.

c. Match each line with its slope using the list below. Note: There are more slopes than lines.

$\frac{\Delta y}{\Delta x} = 6$	$\frac{\Delta y}{\Delta x} = 2$	$\frac{\Delta y}{\Delta x} = -\frac{1}{5}$	$\frac{\Delta y}{\Delta x} = \frac{3}{2}$
$\frac{\Delta y}{\Delta x} = 0$	$\frac{\Delta y}{\Delta x} = -\frac{2}{3}$	$\frac{\Delta y}{\Delta x} = -5$	$\frac{\Delta y}{\Delta x} = \frac{2}{3}$

d. Viewed left to right, in what direction would a line with slope $-\frac{3}{5}$ point? How do you know?

e. Viewed left to right, in what direction would a line with slope $-\frac{5}{3}$ point? How do you know? How would it be different from the line in part (d)?

METHODS AND MEANINGS

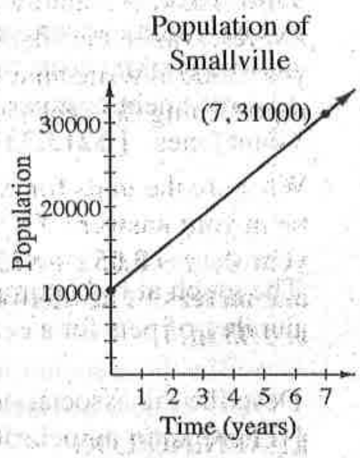
MATH NOTES

Slope as Rate and Dimensional Analysis

Slope as Rate

The slope of a line can represent many things. In the Big Race you concentrated on situations where the rate of change of a line (the slope) represented speed in meters/second. However, rate of change can represent many other things besides speed, depending on the situation.

Since the slope is $\frac{\Delta y}{\Delta x}$, the units of the slope are $\frac{\text{units of } \Delta y}{\text{units of } \Delta x}$. Often, rates of change are written as unit rates (that is, rates with a denominator of 1). In the situation shown in the graph at right, the slope is $\frac{\Delta y}{\Delta x} = \frac{21000 \text{ people}}{7 \text{ years}}$, so the unit rate of change in the population of Smallville is 3000 people/year.



Dimensional Analysis

To use different units for the rate of change (or any other measurement), use dimensional analysis and the conversion factors from the Lesson 2.2.4 Math Notes box. Use the reciprocals of conversion factors as needed to make Giant Ones out of the units.

For example, to calculate the rate of change of the population of Smallville in people per hour, follow the steps at right.

$$\frac{3000 \text{ people}}{1 \text{ year}} \cdot \frac{1 \text{ year}}{365 \text{ days}} \cdot \frac{1 \text{ day}}{24 \text{ hours}}$$

$$= \frac{3000}{1} \cdot \frac{\text{people}}{1} \cdot \frac{1}{365} \cdot \frac{\cancel{\text{year}}}{\cancel{\text{year}}} \cdot \frac{1}{24} \cdot \frac{\cancel{\text{day}}}{\cancel{\text{day}}} \cdot \frac{1}{\text{hours}}$$

$$\approx \frac{0.342 \text{ people}}{\text{hour}}$$

Parent Guide & Extra Practice

DETERMINING POINTS OF INTERSECTION 1.2.1

Multiple representations (table, graph, equation, situation) can be used to determine where the graphs of two functions will intersect. In a table, find the entries with the same x- and y-values. On a graph, you can usually see the point(s) of intersection, although they might not be "nice" coordinates. The equations of two functions can be set equal to each other and solved to determine the exact points of intersection.

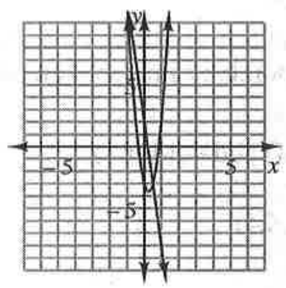
Example

Where do the graph of the functions $f(x) = 10x^2 - 5x - 3$ and $= -10x + 2$ intersect?

Solution: Create tables and a graph for the given functions.

x	$f(x)$
–3	102
–2	47
–1	12
0	–3
1	2
2	27
3	72

x	$g(x)$
–3	32
–2	22
–1	12
0	2
1	–8
2	–18
3	–28



Both table contain the point (–1, 12), so that is one point of intersection. This point of intersection cannot be seen in a standard graphing window.

By looking at the graph we can see that there is another point of intersection, but it does not have integer coordinates. Use the Equal Values Method (from *Core Connections Integrated I*) to solve for this point. Since the graphs of the functions intersect when $f(x) = g(x)$, start by solving the equation $10x^2 - 5x - 3 = -10x + 2$.

$10x^2 - 5x - 3 = -10x + 2$	This is a quadratic equation, so start be setting it equal to 0.
$10x^2 + 5x - 5 = 0$	Move all terms to one side.
$5(2x^2 + x - 1) = 0$	Factor out the greatest common factor.
$5(2x - 1)(x + 1) = 0$	Factor the quadratic expression.
$5 = 0$ or $2x - 1 = 0$ or $x + 1 = 0$	Apply the Zero Product Property.
$5 \neq 0$ so only $x = \frac{1}{2}$ or $x = -1$	Solve each equation.

Solve for the corresponding y-values. It does not matter which equation you use.

If $x = \frac{1}{2}$, then $y = -10\left(\frac{1}{2}\right) + 2$ or $y = -3$. $\left(\frac{1}{2}, -3\right)$ is a point of intersection.

If $x = -1$, then $y = -10(-1) + 2$ or $y = 12$. $(-1, 12)$ is a point of intersection.

So far you have looked at several similar slope triangles and their corresponding slope ratios. These relationships will be very useful for determining missing side lengths or angle measures of right triangles for the rest of this chapter.

Before you forget this valuable information, organize information about the triangles and ratios you have discovered so far in the table on the Lesson 3.2.2 ("Trig Table Graphic Organizer") Resource Page. Keep it in a safe place for future reference. Include all of the angles you have studied up to this point. An example for 11° is filled in on the table to get you started.

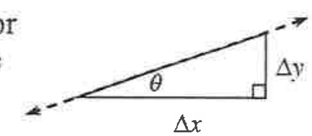
METHODS AND MEANINGS

MATH NOTES

Slope and Angle Notation

The **slope** of a line is the ratio of the vertical distance to the horizontal distance in a slope triangle formed by two points on a line. The vertical part of the triangle is called Δy, (read "change in y"), while the horizontal part of the triangle is called Δx (read "change in x"). Slope can then be written as $\frac{\Delta y}{\Delta x}$. Slope indicates both how steep the line is and its direction, upward or downward, left to right. The Greek capital letter Δ (*Delta*) is often used in mathematics to describe change.

Mathematicians sometimes use Greek letters as variables for angle measurement. The most common variable for an angle is the Greek letter θ (*theta*), pronounced "THAY-tah". Two other Greek letters commonly used include a (*alpha*), and b (*beta*), pronounced "BAY-tah".



When a right triangle is oriented like a slope triangle, such as the one in the diagram above, the angle the line makes with the horizontal side of the triangle is called a **slope angle**.

SAT PREP

1. If $7x < 2y$ and $2y < 9z$, which of the following statements is true?

   a. $7x < 9z$ b. $9z < 7x$ c. $z < x$ d. $7x = 9z$ e. $7x + 1 = 9z$

2. If $f(t) = 5t - 15$, then at what value of t does the graph of $y = f(t)$ cross the x-axis?

   a. –15 b. –5 c. 0 d. 2 e. 3

3. If $p^5 + 3 = p^5 + w$, then w = ?

   a. –3 b. $-\sqrt[5]{3}$ c. $\sqrt[5]{3}$ d. 3 e. 3^5

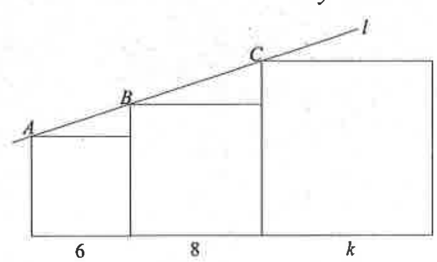
4. For all positive numbers j and k, let $j\nabla k$ be defined as $\frac{j+4k}{j-4k}$. What is the value of $1{,}018\nabla 4.5$?

   a. 1.036 b. 10.36 c. 103.6 d. 1036 e. 10360

5. If a number is rounded to 26.7, which of the following values could have been the original number?

   a. 26 b. 26.605 c. 26.655 d. 26.776 e. 27

6. On a coordinate plane, the center of a circle is at (9 ,–2). If the circle touches the y-axis in only one point, what is the radius of the circle?



7. The figure at right shows three squares with sides of length 6, 8, and k, respectively. If points A, B, and C lie on line l, what is the value of k?

8. Exactly 875 out of 7000 seniors at college are majoring in mathematics. What percent of seniors are NOT majoring in mathematics?

9. Five SnookerBars cost as much as 2 Sodiepop Swirls. If the cost of one Sodiepop Swirl and one SnookerBar is $1.75, what is the cost in dollars of 1 Sodiepop Swirl?

10. The highest score possible on Professor Snape's test is 100 and the lowest is 0. Harry, Ron, Hermione, and Neville's tests had an average of 86. If Neville got the lowest score, what is the lowest possible score he could have gotten?

Answers

1.	A	2.	E	3.	D	4.	A	5.	C
6.	9	7.	$\frac{32}{3}$	8.	87.5%	9.	$1.25	10.	4